

Weak cosmic censorship, dyonic Kerr–Newman black holes and Dirac fields

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Abstract

It was investigated recently, with the aim of testing the weak cosmic censorship conjecture, whether an extremal Kerr black hole can be converted into a naked singularity by interaction with a massless classical Dirac test field, and it was found that this is possible. We generalize this result to electrically and magnetically charged rotating extremal black holes (i.e. extremal dyonic Kerr–Newman black holes) and massive Dirac test fields, allowing magnetically or electrically uncharged or nonrotating black holes and the massless Dirac field as special cases. We show that the possibility of the conversion is a direct consequence of the fact that the Einstein–Hilbert energy-momentum tensor of the classical Dirac field does not satisfy the null energy condition, and is therefore not in contradiction with the weak cosmic censorship conjecture. We give a derivation of the absence of superradiance of the Dirac field without making use of the complete separability of the Dirac equation in dyonic Kerr–Newman background, and we determine the range of superradiant frequencies of the scalar field. The range of frequencies of the Dirac field that can be used to convert a black hole into a naked singularity partially coincides with the superradiant range of the scalar field. We apply horizon-penetrating coordinates, as our arguments involve calculating quantities at the event horizon. We describe the separation of variables for the Dirac equation in these coordinates, although we mostly avoid using it.

1 Introduction

The well-known weak cosmic censorship conjecture (WCCC), stated originally by Penrose [1], asserts that naked singularities (i.e. gravitational singularities not hidden behind an event horizon) generically cannot be produced in a physical process from regular initial conditions, if the matter involved in the process has reasonable properties. Although there is significant evidence in favour of the validity of this conjecture, finding a general proof remains one of the major unsolved problems of classical general relativity. (For a more detailed and precise description of the WCCC and for reviews on results regarding its validity see [2, 3, 4, 5, 6, 7].)

As long as a complete proof is not available, it is interesting to test the WCCC in various special cases. A possible such test is a thought experiment in which a small particle is thrown at a Kerr–Newman black hole and it is checked if an overextremal Kerr–Newman spacetime, which contains a naked singularity, can arise after the particle has been absorbed by the black hole. This thought experiment was considered first in [8], where it was shown that an extremal Kerr–Newman black hole cannot be overcharged or overspun by throwing a pointlike test particle with electric charge into it. In particular, it was shown that if a particle has a charge or angular momentum that would make the black hole overextremal if it absorbed the particle, then the particle will not fall into the black hole. A simpler derivation of this result was given in [9]. In [10] and [11] the result of [8] was extended to dyonic Kerr–Newman black holes, which are rotating black holes with both electric and magnetic charge. More recently another version of the thought experiment in which various test fields (scalar, electromagnetic and Dirac) are used instead of point particles was also considered [12, 13, 14, 15, 16]. It was found that the weak cosmic censorship is not violated in these cases either, with the exception of the case when the test field is a Dirac field [15]. Such a result is not surprising, since the WCCC is expected to be valid only for matter that has “reasonable” properties, among which a suitable energy condition is included (see e.g. [2, 3]), and the Dirac field is well known not to satisfy the weak energy condition [17], in contrast with the scalar and electromagnetic fields. Studying the case of Dirac test fields is interesting, nevertheless, because fermionic matter has an important role in physics.

In the present paper we extend the result of [15], which applies to Kerr black holes and massless neutral Dirac fields, to charged rotating black holes and charged massive Dirac fields. For the sake of generality we allow the black hole to have magnetic charge as well, i.e. we consider dyonic Kerr–Newman black holes, but we stress that the cases of Kerr–Newman, Reissner–Nordström and Kerr black holes and neutral or massless Dirac fields can be obtained from the general case by suitable special choice of the parameters.

The arguments in this paper are technically different from [15] in a few aspects. First, we make little use of the complete separability of the Dirac equation in dyonic Kerr–Newman background; we mainly use only Fourier expansion in the time and azimuthal angle variables, along with simple properties of the Dirac field. Second, we apply horizon-penetrating coordinates, since these are well suited for calculating

fluxes at the event horizon. Third, instead of the Newman–Penrose formalism we use orthonormal tetrads and four-component Dirac spinor formalism. This is done to keep the formalism close to the usual Minkowski spacetime formulation of Dirac fields (see e.g. [81]). Fourth, we construct the energy and angular momentum currents using Noether’s theorem rather than the Einstein–Hilbert energy-momentum tensor, because the latter method is not suitable in the presence of external electromagnetic fields.

The Dirac field has another remarkable feature in which it differs from the scalar and electromagnetic fields, namely it does not exhibit superradiance in black hole spacetimes. After discussing the thought experiment we present a derivation of this result as well, because it requires arguments similar to those used for the thought experiment, and because the derivations that can be found in the literature usually apply the complete separability of the Dirac equation (see e.g. [17, 18, 19, 20, 21, 22, 23]), but we would like to emphasize that this is not necessary. Our derivation is similar to the one that is outlined in [2, 24]. Moreover, the non-superradiant nature of the Dirac field is also related to its property that it does not satisfy the weak energy condition (see e.g. [17, 18, 2]). We determine the superradiant frequency range of the scalar field as well, because it has relevance for the thought experiment. The superradiance of the scalar field is discussed in several articles (see e.g. [2, 18, 33]), but usually at zero magnetic charge, and often in a way that relies on the complete separability of the field equation.

The paper is organized as follows. In Section 2 the Dirac field is introduced and its conservation laws relevant for the thought experiment are discussed. This is done in a general setting, i.e. the discussion is not specialized to black hole spacetimes. In Section 3 the relevant properties of dyonic Kerr–Newman black holes are recalled. In Section 4 the thought experiment is described and the derivation of the main result, which indicates the possibility of the formation of a naked singularity as a result of the interaction of a black hole and a classical Dirac field, is presented. A discussion of the relevance of backreaction effects is also included. In Section 5 the absence of superradiance of Dirac fields around dyonic Kerr–Newman black holes is derived and the superradiant frequency range of the scalar field is determined. Conclusions are given in Section 6. In A a part of the formalism of spinor fields in curved spacetime is recalled for completeness and to fix notation. In B the separation of variables for the Dirac equation, pertaining to the horizon-penetrating coordinates and to the tetrad used in this paper, is described. The asymptotic behaviour of the radial functions at the event horizon is also determined.

The signature of metric tensors will be $(+, -, -, -)$.

2 The Dirac field

The Lagrangian density of the Dirac field Ψ in fixed gravitational and electromagnetic fields is

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}[\bar{\Psi}i\gamma_{\mu}(\nabla_{\nu} + ieA_{\nu})\Psi - (\nabla_{\nu} - ieA_{\nu})\bar{\Psi}i\gamma_{\mu}\Psi] - m\bar{\Psi}\Psi, \quad (2.1)$$

where A_μ is the vector potential of the electromagnetic field, m is the mass parameter of the Dirac field and e is the electromagnetic coupling constant. For the definition of ∇_μ , γ_μ and $\bar{\Psi}$ see A. The Euler–Lagrange equation corresponding to \mathcal{L} is the Dirac equation, $i\gamma^\mu(\nabla_\mu + ieA_\mu)\Psi = m\Psi$.

2.1 Conserved currents

The electric current of the Dirac field is

$$j_{em}^\mu = -e\bar{\Psi}\gamma^\mu\Psi = \frac{\partial\mathcal{L}}{\partial A_\mu} . \quad (2.2)$$

The closely related current $j^\mu = \bar{\Psi}\gamma^\mu\Psi$ is often called particle number density current. The vector j^μ has the important and well known property that it is real, future directed and time-like or null for any Dirac spinor Ψ , regardless of the equation of motion. Furthermore, one can also verify that $(\bar{\Psi}\gamma_\mu\Psi)(\bar{\Psi}\gamma^\mu\Psi) = 4w^*w$, where $w = \Psi_1^*\Psi_3 + \Psi_2^*\Psi_4$, thus j^μ is null if and only if $w = 0$. These properties of j^μ imply that the electric charge of a classical Dirac field has a definite sign, which is the same as the sign of $-e$.

Regarding conserved currents associated with Killing fields, a standard way in general relativity to construct such currents is to take $\mathbf{T}^{\mu\nu}K_\nu$, where K^μ is the relevant Killing vector field and $\mathbf{T}^{\mu\nu}$ is the Einstein–Hilbert energy-momentum tensor obtained by the variation of the matter action with respect to the metric. The conservation of $\mathbf{T}^{\mu\nu}K_\nu$ follows from $\nabla_\mu\mathbf{T}^{\mu\nu} = 0$ and from the Killing equation. Although the Lagrangian density (2.1) depends explicitly (i.e. not only through the metric) on the tetrad field, the definition of the Einstein–Hilbert energy-momentum tensor can be extended to such cases (see e.g. [80]). However, as is well known, in the presence of external fields (in particular in the presence of an external electromagnetic field) generally $\nabla_\mu\mathbf{T}^{\mu\nu} \neq 0$ and $\mathbf{T}^{\mu\nu}K_\nu$ is not conserved, thus one has to find some other way to construct a suitable conserved current. If the matter action is invariant under the diffeomorphisms generated by the Killing field, then Noether’s theorem is still available for this purpose. In the following we discuss the Noether currents of the Dirac field associated with Killing fields, and compare them with the currents $\mathbf{T}^{\mu\nu}K_\nu$.

Let us assume that coordinates are chosen so that there is one coordinate function, which we denote by t , for which $K^\mu = (\partial_t)^\mu$. In these coordinates K^μ generates translations of t . Let us also assume that the tetrad (and thus also γ^μ) is chosen so that it is invariant under t -translations. In addition, the vector potential of the external electromagnetic field is also assumed to be invariant under t -translations. In this case the action of the Dirac field is invariant under t -translations, and the straightforward application of Noether’s theorem gives the conserved current

$$\mathcal{E}^\mu = \frac{\partial\mathcal{L}}{\partial_\mu\Psi}\partial_t\Psi + \frac{\partial\mathcal{L}}{\partial_\mu\bar{\Psi}}\partial_t\bar{\Psi} - \delta_t^\mu\mathcal{L} = \frac{1}{2}(i\bar{\Psi}\gamma^\mu\partial_t\Psi - i\partial_t\bar{\Psi}\gamma^\mu\Psi) . \quad (2.3)$$

On the right hand side the term $\delta_t^\mu\mathcal{L}$ is omitted because $\mathcal{L} = 0$ if Ψ satisfies the Dirac equation. It is worth noting that \mathcal{E}^μ is real, and if Ψ has the t -dependence $\Psi = e^{-i\omega t}\psi$, then $\mathcal{E}^\mu = \omega j^\mu$.

If the electromagnetic field or e is zero, then \mathbf{T}^μ_t is also conserved, thus it is natural to ask what the relation between \mathbf{T}^μ_t and \mathcal{E}^μ is in this case. In the following we show that the answer to this question is that the difference between these two currents is a current of the form $\nabla_\nu f^{\mu\nu}$, where $f^{\mu\nu}$ is antisymmetric, therefore \mathbf{T}^μ_t and \mathcal{E}^μ can be considered to be equivalent. In fact we derive a more general result, equation (2.8), which holds also in the presence of electromagnetic field. (2.8) will be useful in Section 4.

The Einstein–Hilbert energy-momentum tensor of the Dirac field is

$$\begin{aligned} \mathbf{T}^{\mu\nu} = & \frac{1}{4} \left(\bar{\Psi} i \gamma^\mu (\nabla^\nu + i e A^\nu) \Psi + \bar{\Psi} i \gamma^\nu (\nabla^\mu + i e A^\mu) \Psi \right. \\ & \left. - (\nabla^\mu - i e A^\mu) \bar{\Psi} i \gamma^\nu \Psi - (\nabla^\nu - i e A^\nu) \bar{\Psi} i \gamma^\mu \Psi \right). \end{aligned} \quad (2.4)$$

We also introduce the similar tensor

$$\hat{T}_{\mu\nu} = \frac{1}{2} \left(\bar{\Psi} i \gamma_\mu (\partial_\nu + i e A_\nu) \Psi - (\partial_\nu - i e A_\nu) \bar{\Psi} i \gamma_\mu \Psi \right), \quad (2.5)$$

which will appear in Section 4 as well, and we define $f^{\mu\nu}$ as

$$f^{\mu\nu} = -\frac{1}{8} i \bar{\Psi} (\gamma^\mu \gamma_t \gamma^\nu - \gamma^\nu \gamma_t \gamma^\mu) \Psi. \quad (2.6)$$

By evaluating $\nabla_\nu f^{\mu\nu}$ one finds that if Ψ satisfies the Dirac equation, then

$$\begin{aligned} \nabla_\nu f^{\mu\nu} = & \frac{1}{4} [i \bar{\Psi} \gamma^\mu \nabla_t \Psi - i \bar{\Psi} \gamma_t \nabla^\mu \Psi - i \nabla_t \bar{\Psi} \gamma^\mu \Psi + i \nabla^\mu \bar{\Psi} \gamma_t \Psi] \\ & - \frac{1}{2} [i \bar{\Psi} \gamma^\mu (\nabla_t - \partial_t) \Psi - i (\nabla_t - \partial_t) \bar{\Psi} \gamma^\mu \Psi] \\ & + \frac{1}{2} e A^\mu \bar{\Psi} \gamma_t \Psi - \frac{1}{2} e A_t \bar{\Psi} \gamma^\mu \Psi. \end{aligned} \quad (2.7)$$

From this result and from (2.4) and (2.5), it can be seen immediately that

$$\hat{T}^\mu_t - \mathbf{T}^\mu_t = \nabla_\nu f^{\mu\nu}. \quad (2.8)$$

The current on the right hand side is conserved for arbitrary Ψ , because $f^{\mu\nu}$ is by definition antisymmetric.

By applying Stokes's theorem it is easy to show, and is well known, that if a current has the form $\nabla_\nu f^{\mu\nu}$, where $f^{\mu\nu}$ is antisymmetric, then any corresponding charge associated with some hypersurface (which does not need to be space-like) is zero if the surface integral arising in the application of Stokes's theorem vanishes. Therefore in view of (2.8) \hat{T}^μ_t and \mathbf{T}^μ_t can be considered to be equivalent.

In the absence of electromagnetic field $\hat{T}^\mu_t = \mathcal{E}^\mu$, thus in this case (2.8) shows that \mathcal{E}^μ and \mathbf{T}^μ_t are equivalent.

We note that a similar but more special result on the equivalence of \mathcal{E}^μ and \mathbf{T}^μ_t can be found in [31].

3 The dyonic Kerr–Newman black holes

A dyonic Kerr–Newman black hole can be characterized by four parameters, the mass M , the angular momentum per unit mass a , the electric charge Q_e and the magnetic charge Q_m . The angular momentum of the black hole is $J = aM$, and $Q_m = 0$ corresponds to a usual Kerr–Newman black hole. The metric of the dyonic Kerr–Newman black hole spacetime with parameters (M, a, Q_e, Q_m) is the same as the Kerr–Newman metric with parameters (M, a, q) , $q^2 = Q_e^2 + Q_m^2$, where q denotes the electric charge parameter of the Kerr–Newman metric. The parameters have to satisfy the inequality

$$\eta = M^2 - Q_e^2 - Q_m^2 - a^2 \geq 0, \quad (3.1)$$

otherwise the spacetime contains a naked singularity. The black hole is called extremal if $\eta = 0$. Under certain conditions, the dyonic Kerr–Newman black holes are the only static and asymptotically flat black hole solutions of the Einstein–Maxwell equations [34, 35].

The vector potential of the electromagnetic field of a dyonic Kerr–Newman black hole is

$$A = Q_e A_e + Q_m A_m, \quad (3.2)$$

where

$$A_e = -\frac{r}{\Sigma} dt + \frac{ar \sin^2 \theta}{\Sigma} d\phi \quad (3.3)$$

$$A_m = \frac{a \cos \theta}{\Sigma} dt + \left[\tilde{C} - \frac{r^2 + a^2}{\Sigma} \cos \theta \right] d\phi, \quad (3.4)$$

$$\Sigma = r^2 + a^2 \cos^2 \theta. \quad (3.5)$$

These formulas are written in Boyer–Lindquist coordinates (t, r, θ, ϕ) . The electromagnetic field derived from A_m is dual to the electromagnetic field derived from A_e . The electromagnetic field does not depend on the constant \tilde{C} , which can be used, by setting $\tilde{C} = 1$ or $\tilde{C} = -1$, to eliminate the Dirac string singularity of A_m along the positive or negative z axis ($\theta = 0$ and $\theta = \pi$), respectively. We set \tilde{C} to zero for a reason that is explained below.

3.1 Horizon-penetrating coordinates

In the following sections various quantities will be considered at the future event horizon. Since the Boyer–Lindquist coordinates do not cover the future event horizon, Eddington–Finkelstein-type ingoing horizon-penetrating coordinates, denoted by $(\tau, r, \theta, \varphi)$, will be used. These coordinates can be introduced by the transformation

$$\tau = t - r + \int dr \frac{r^2 + a^2}{\Delta}, \quad \varphi = \phi + \int dr \frac{a}{\Delta}, \quad (3.6)$$

where $\Delta = r^2 + a^2 + Q_e^2 + Q_m^2 - 2Mr$. The future event horizon is located in these coordinates at the constant value $r_+ = M + \sqrt{M^2 - (a^2 + Q_e^2 + Q_m^2)}$ of r , and

the metric is non-singular in these points. The inner horizon is located at $r_- = M - \sqrt{M^2 - (a^2 + Q_e^2 + Q_m^2)}$. In the extremal case $r_+ = r_- = M$. The $(\tau + r, \theta, \varphi) = \text{constant}$ lines are ingoing null geodesics, and there exists an $r_0 < r_+$ such that the $\tau = \text{constant}$ hypersurfaces are space-like in the domain $r_0 < r$.

The r component $(A_e)_r$ of A_e with respect to the coordinates $(\tau, r, \theta, \varphi)$ is singular at the event horizon, but this singularity can be eliminated by the gauge transformation $A_e \rightarrow A_e - \frac{r}{\Delta} dr$. After this gauge transformation

$$A_e = -\frac{r}{\Sigma} d\tau + \frac{ar \sin^2 \theta}{\Sigma} d\varphi - \frac{r}{\Sigma} dr . \quad (3.7)$$

The r component of A_m with respect to the coordinates $(\tau, r, \theta, \varphi)$ is also singular if $\tilde{C} \neq 0$, therefore we set $\tilde{C} = 0$. Nevertheless, in order to treat the Dirac string singularity of A_m , we introduce an explicit gauge parameter into it by adding $C d\varphi$, where C is a real constant. Thus

$$A_m = \frac{a \cos \theta}{\Sigma} d\tau + \left[C - \frac{r^2 + a^2}{\Sigma} \cos \theta \right] d\varphi + \frac{a \cos \theta}{\Sigma} dr . \quad (3.8)$$

Generally A_m has a string singularity along the z axis (which corresponds to $\theta = 0$ and $\theta = \pi$) because $d\varphi$ is singular here, and its coefficient $(A_m)_\varphi$ does not cancel this singularity. However, in the special cases $C = 1$ and $C = -1$ the singularity is cancelled along the positive z axis ($\theta = 0$) or along the negative z axis ($\theta = \pi$), respectively. The string singularity can therefore be avoided by using two domains that cover the whole spacetime region of interest in such a way that one of the domains contains the entire positive z axis but is well separated from the negative z axis and the other one contains the entire negative z axis but is separated from the positive z axis. In the first domain the $C = 1$ gauge is used then, and in the second domain the $C = -1$ gauge. Suitable domains are given by the relations $r_0 < r$, $0 \leq \theta < \pi/2 + \epsilon$ and $r_0 < r$, $\pi/2 - \epsilon < \theta \leq \pi$, where ϵ is some small number. These domains will be denoted by \mathcal{D}_+ and \mathcal{D}_- . It should be kept in mind that the transition between the two domains involves a gauge transformation. This approach to treating the string singularity of A_m was proposed in [36] and was taken also in [11, 12, 16].

In the following sections and in B, except in Section 3.3, we use only the coordinates $(\tau, r, \theta, \varphi)$, and we also use the notation ζ for the one-form dr (the exterior derivative of the coordinate function r), i.e.

$$\zeta^\mu = (dr)^\mu . \quad (3.9)$$

A_e , A_m and A will denote (3.7), (3.8) and $A = Q_e A_e + Q_m A_m$, respectively.

3.2 Various important properties

In this section further important properties of the Dyonic Kerr–Newman black holes, which will be used in the subsequent sections, are collected.

∂_τ and ∂_φ are Killing fields; ∂_τ is the generator of time translations and ∂_φ is the generator of rotations around the axis of the black hole. A_e and A_m are also invariant under these symmetries. The Killing field

$$\chi = \partial_\tau + \Omega_H \partial_\varphi, \quad \Omega_H = \frac{a}{r_+^2 + a^2} \quad (3.10)$$

is null at the event horizon. In the subsequent sections it will also be important that at the event horizon

$$(A_e)_\mu \chi^\mu = \frac{-r_+}{r_+^2 + a^2}, \quad (A_m)_\mu \chi^\mu = C \Omega_H, \quad (3.11)$$

and ζ^μ is parallel to χ^μ . The relation between ζ^μ and χ^μ at the event horizon is

$$\zeta^\mu = -\frac{r_+^2 + a^2}{r_+^2 + a^2 \cos^2 \theta} \chi^\mu, \quad (3.12)$$

thus ζ^μ is past directed (in [16] ζ^μ was denoted by ω^μ and it was future directed because of the opposite signature of the metric there). (3.12) shows that ζ^μ is null at the event horizon, but it should be stressed that this property of ζ^μ follows directly from the facts that the event horizon is a null surface and is a level surface of the function r .

It is useful to introduce the quantity Φ_H as

$$\Phi_H = \frac{r_+ Q_e}{r_+^2 + a^2}. \quad (3.13)$$

In the case of Kerr–Newman black holes, Φ_H is known as the electrostatic potential of the horizon.

3.3 Tetrad

In order to define a suitable tetrad for the Kerr–Newman metric one can start with the Kinnersley-type tetrad (see also [37])

$$V_\mu^{\bar{0}} = \frac{1}{\sqrt{2}} \left(\left(1 + \frac{\Delta}{2\Sigma} \right) dt + \left(\frac{1}{2} - \frac{\Sigma}{\Delta} \right) dr - \left(1 + \frac{\Delta}{2\Sigma} \right) a \sin^2 \theta d\phi \right) \quad (3.14)$$

$$V_\mu^{\bar{1}} = -\frac{a^2 \cos \theta \sin \theta}{\Sigma} dt + r d\theta + \frac{a(a^2 + r^2) \cos \theta \sin \theta}{\Sigma} d\phi \quad (3.15)$$

$$V_\mu^{\bar{2}} = \frac{ar \sin \theta}{\Sigma} dt + a \cos \theta d\theta - \frac{r(a^2 + r^2) \sin \theta}{\Sigma} d\phi \quad (3.16)$$

$$V_\mu^{\bar{3}} = \frac{1}{\sqrt{2}} \left(\left(-1 + \frac{\Delta}{2\Sigma} \right) dt + \left(\frac{1}{2} + \frac{\Sigma}{\Delta} \right) dr + \left(1 - \frac{\Delta}{2\Sigma} \right) a \sin^2 \theta d\phi \right), \quad (3.17)$$

given in Boyer–Lindquist coordinates. This can be transformed into the ingoing horizon-penetrating coordinates, but one finds that it is singular at the event horizon.

This singularity can nevertheless be removed by a suitable local Lorentz transformation, similarly as for example in [32]. Thus in the present paper we use the Lorentz transformed non-singular tetrad $\tilde{V}_\mu^{\bar{\mu}}$ related to $V_\mu^{\bar{\mu}}$ as

$$\tilde{V}_\mu^{\bar{0}} = \frac{r^2}{\Delta}(V_\mu^{\bar{0}} + V_\mu^{\bar{3}}) + \frac{\Delta}{r^2}(V_\mu^{\bar{0}} - V_\mu^{\bar{3}}) \quad (3.18)$$

$$\tilde{V}_\mu^{\bar{3}} = \frac{r^2}{\Delta}(V_\mu^{\bar{0}} + V_\mu^{\bar{3}}) - \frac{\Delta}{r^2}(V_\mu^{\bar{0}} - V_\mu^{\bar{3}}) \quad (3.19)$$

$$\tilde{V}_\mu^{\bar{1}} = V_\mu^{\bar{1}}, \quad \tilde{V}_\mu^{\bar{2}} = V_\mu^{\bar{2}}. \quad (3.20)$$

$\tilde{V}_\mu^{\bar{\mu}}$ and $V_\mu^{\bar{\mu}}$ are invariant under time translations and under rotations around the axis of the black hole, and $\tilde{V}_\mu^{\bar{\mu}}$ tends to $V_\mu^{\bar{\mu}}$ if $r \rightarrow \infty$. It should also be mentioned that $V_\mu^{\bar{\mu}}$ and $\tilde{V}_\mu^{\bar{\mu}}$ are not null tetrads, rather $V_\mu^{\bar{\mu}}V^{\mu\bar{\nu}} = \tilde{V}_\mu^{\bar{\mu}}\tilde{V}^{\mu\bar{\nu}} = g^{\bar{\mu}\bar{\nu}}$, where $g^{\bar{\mu}\bar{\nu}} = \text{diag}(1, -1, -1, -1)$. We note finally that another useful choice for $V_\mu^{\bar{\mu}}$ would be the ‘canonical’ tetrad of Carter [37, 38].

4 The thought experiment

The thought experiment for testing the WCCC is assumed to proceed in the following way. Initially one has an extremal dyonic Kerr–Newman black hole, then a small amount of matter represented by a wave packet is thrown at it from great distance. A certain part of the matter is absorbed by the black hole, the remaining part is scattered back to infinity, and finally the system settles down in another dyonic Kerr–Newman state with slightly different parameters.

Under an infinitesimally small change (dM, dJ, dQ_e, dQ_m) of the parameters (M, J, Q_e, Q_m) of a dyonic Kerr–Newman configuration the change of η (which was introduced in (3.1)) is

$$d\eta = 2\frac{M^2 + a^2}{M} \left(dM - \frac{a}{M^2 + a^2}dJ - \frac{Q_e M}{M^2 + a^2}dQ_e - \frac{Q_m M}{M^2 + a^2}dQ_m \right). \quad (4.1)$$

If one calculates the change (dM, dJ, dQ_e, dQ_m) of the parameters in the process described above, one should find $d\eta \geq 0$, if the final state is a dyonic Kerr–Newman black hole and cosmic censorship is not violated, whereas a result $d\eta < 0$ indicates the formation of a naked singularity, and thus a violation of the WCCC. Of course, in the case $d\eta < 0$ the last conclusion that the WCCC is violated can be drawn only if the matter used in the thought experiment does have the properties required in the WCCC.

In the calculation of (dM, dJ, dQ_e, dQ_m) the test matter approximation is used, i.e. the metric and the electromagnetic field are considered fixed and backreaction effects are neglected. The reason for taking the initial black hole state to be extremal is that the quantities (dM, dJ, dQ_e, dQ_m) are very small, in accordance with the test matter approximation.

There are several articles, e.g. [39]-[71], in which other versions or aspects of the thought experiment are studied. For instance, backreaction effects and subextremal initial black holes are considered in several papers. Furthermore, besides the thought experiment it is interesting to study the possibilities of observing naked singularities that may form if the WCCC is violated; see e.g. [74]-[79].

We turn now to the calculation of $d\eta$. In the following the black hole is not restricted to be extremal unless explicitly stated. Applying (2.3) to the Killing fields $(\partial_\tau)^\mu$ and $(\partial_\varphi)^\mu$ one obtains that the energy and angular momentum currents are given by the equations

$$\mathcal{E}^\mu = \hat{T}^\mu_\tau + eA_\tau j^\mu \quad (4.2)$$

$$\mathcal{J}^\mu = \hat{T}^\mu_\varphi + eA_\varphi j^\mu, \quad (4.3)$$

where $\hat{T}_{\mu\nu}$ is given by (2.5).

$\hat{T}_{\mu\nu}$ and j^μ are gauge invariant and A_τ does not depend on the gauge parameter C , therefore \mathcal{E}^μ is also independent of C . A_φ does depend on C , however, thus \mathcal{J}^μ also depends on it. For this reason we take (as in [16]) the modified definition

$$\mathcal{J}^\mu = \hat{T}^\mu_\varphi + e(A_\varphi - Q_m C)j^\mu \quad (4.4)$$

for \mathcal{J}^μ , which eliminates its dependence on C . The conservation of \mathcal{J}^μ is not affected by this modification, because j^μ is conserved. The independence of \mathcal{E}^μ and \mathcal{J}^μ of C is important because the value of C is different in the domains \mathcal{D}_+ and \mathcal{D}_- .

The electric charge flux through the event horizon into the black hole is

$$\frac{dQ}{d\tau} = \int_H \sqrt{-g} \, e j^r \, d\theta d\varphi, \quad (4.5)$$

where H denotes the two-dimensional surface of the black hole (which is the relevant time slice of the event horizon), and the energy and angular momentum fluxes are

$$\frac{dE}{d\tau} = - \int_H \sqrt{-g} \left[\hat{T}^r_\tau + eA_\tau j^r \right] d\theta d\varphi \quad (4.6)$$

$$\frac{dL}{d\tau} = \int_H \sqrt{-g} \left[\hat{T}^r_\varphi + e(A_\varphi - Q_m C)j^r \right] d\theta d\varphi, \quad (4.7)$$

where the quantities in the brackets are \mathcal{E}^r and \mathcal{J}^r , respectively. The total energy, angular momentum and electric charge that falls through the event horizon is $\int_{-\infty}^{\infty} \frac{dE}{d\tau} d\tau$, $\int_{-\infty}^{\infty} \frac{dL}{d\tau} d\tau$ and $\int_{-\infty}^{\infty} \frac{dQ}{d\tau} d\tau$, respectively. The metric and the electromagnetic field are taken to be fixed, therefore these quantities can be identified with dM , dJ and dQ_e , i.e. with the change of the mass, angular momentum and electric charge of the black hole. $dQ_m = 0$, since the Dirac field does not have magnetic charge.

From the equations (4.5), (4.6), (4.7) above and from (3.10), (3.11) and (3.13) it follows immediately that

$$- \int_H \sqrt{-g} \, \hat{T}_{\mu\nu} \zeta^\mu \chi^\nu \, d\theta d\varphi = \frac{dE}{d\tau} - \Omega_H \frac{dL}{d\tau} - \Phi_H \frac{dQ}{d\tau}. \quad (4.8)$$

Taking into account the relations $dM = \int_{-\infty}^{\infty} \frac{dE}{d\tau} d\tau$, $dJ = \int_{-\infty}^{\infty} \frac{dL}{d\tau} d\tau$ and $dQ_e = \int_{-\infty}^{\infty} \frac{dQ}{d\tau} d\tau$,

$$- \int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} \hat{T}_{\mu\nu} \zeta^\mu \chi^\nu d\theta d\varphi = dM - \Omega_H dJ - \Phi_H dQ_e \quad (4.9)$$

is obtained from (4.8). It is easy to see that in the extremal case the right hand side in (4.9) is $\frac{M}{2(M^2+a^2)} d\eta$, thus the sign of $d\eta$ depends, in the extremal case, on the sign of $\int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} \hat{T}_{\mu\nu} \zeta^\mu \chi^\nu d\theta d\varphi$.

In order to examine $\int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} \hat{T}_{\mu\nu} \zeta^\mu \chi^\nu d\theta d\varphi$ it is useful to consider the Fourier expansion

$$\Psi = \sum_n \int d\omega e^{-i\omega\tau} e^{i(n-CeQ_m)\varphi} \psi_{\omega,n}(r, \theta) \quad (4.10)$$

of Ψ , where $e^{-i\omega\tau} e^{i(n-CeQ_m)\varphi} \psi_{\omega,n}(r, \theta)$ are solutions of the Dirac equation. The term $-CeQ_m$ in the factor $e^{i(n-CeQ_m)\varphi}$ is included because of the gauge transformation done at equation (3.8) (see also [37]). For $e^{-i\omega\tau} e^{i(n-CeQ_m)\varphi} \psi_{\omega,n}(r, \theta)$ to be single valued for both $C = 1$ and $C = -1$, both $n - eQ_m$ and $n + eQ_m$ have to be integer, implying that n and eQ_m are either integer or half-integer. The summation in (4.10) should therefore be done over \mathbb{Z} if eQ_m is integer and over $\frac{1}{2} + \mathbb{Z}$ if eQ_m is half-integer. Far from the black hole only modes with $|\omega| > m$ describe propagating waves. Using (4.10) one finds that

$$\begin{aligned} \int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} \hat{T}_{\mu\nu} \zeta^\mu \chi^\nu d\theta d\varphi &= \\ &= (2\pi)^2 \sum_n \int d\omega \int_H d\theta \sqrt{-g} \zeta^\mu \bar{\psi}_{\omega,n} \gamma_\mu \psi_{\omega,n} \left(\omega - n\Omega_H + \frac{eQ_e r_+}{r_+^2 + a^2} \right). \end{aligned} \quad (4.11)$$

In the derivation of (4.11) the only derivatives of Ψ that appear are $\partial_\tau \Psi$ and $\partial_\varphi \Psi$, which are easy to evaluate, and formulas (3.10) and (3.11) can also be applied.

As was mentioned in Section 3.2, ζ^μ is a past directed null vector at the event horizon, and in Section 2.1 it was also noted that $\bar{\psi}_{\omega,n} \gamma_\mu \psi_{\omega,n}$ is a real future directed null or time-like vector, therefore $\zeta^\mu \bar{\psi}_{\omega,n} \gamma_\mu \psi_{\omega,n} \leq 0$. The integrand on the right hand side of (4.11) is thus positive if

$$\tilde{\omega} = \omega - n\Omega_H + \frac{eQ_e r_+}{r_+^2 + a^2} < 0 \quad (4.12)$$

and $\zeta^\mu \bar{\psi}_{\omega,n} \gamma_\mu \psi_{\omega,n} \neq 0$ at the event horizon. (Here the notation $\tilde{\omega}$ has been introduced.) If $\zeta^\mu \bar{\psi}_{\omega,n} \gamma_\mu \psi_{\omega,n} \tilde{\omega}$ is large at the event horizon mainly for those values of ω and n for which $\tilde{\omega} < 0$, then it is possible for the whole integral (4.11) to be positive. In this case $dM - \Omega_H dJ - \Phi_H dQ_e < 0$, in particular in the extremal case $d\eta < 0$, indicating a possible violation of the WCCC.

Clearly $\tilde{\omega}$ is negative if ω has a sufficiently large negative value, but, more interestingly, $\tilde{\omega} < 0$ is possible even for $\omega > 0$, if

$$n\Omega_H - \frac{eQ_e r_+}{r_+^2 + a^2} > 0. \quad (4.13)$$

It is also interesting to note that the frequency range where $\tilde{\omega} < 0$ partially coincides with the range where the scalar field exhibits superradiance (see Section 5.2).

In the special case when the charges Q_e and Q_m of the black hole are zero, one can use instead of (4.2) and (4.3) the energy and angular momentum currents obtained from the Einstein–Hilbert energy-momentum tensor, as is usually done in the literature (see, for example, [15, 17]). In view of the arguments in the last part of Section 2.1, this would give the same result (namely -1 times the right hand side of (4.11)) for $dM - \Omega_H dJ - \Phi_H dQ_e$.

A tensor similar to $\hat{T}^{\mu\nu}$ appears also in that version of the thought experiment in which the test field is a scalar field (see Section 4.1 of [16] and Section 5.2). In that case $\hat{T}^{\mu\nu}$ satisfies the null energy condition $\hat{T}^{\mu\nu}\chi_\mu\chi_\nu \geq 0$ at the event horizon, and this implies that the WCCC is not violated. If this null energy condition held in the case of the Dirac test field, then it could be used in the same way as in the case of the scalar test field to show that $dM - \Omega_H dJ - \Phi_H dQ_e \geq 0$ and the WCCC is not violated.

In the case of the scalar field $\hat{T}^{\mu\nu}$ is the Einstein–Hilbert energy-momentum tensor, and it was shown in the last part of Section 2.1 that $\hat{T}^{\mu\nu}$ is related to the Einstein–Hilbert energy-momentum tensor also in the case of the Dirac field. Moreover, $\hat{T}_{\mu\nu}\zeta^\mu\chi^\nu = \hat{T}^r_\tau + \Omega_H \hat{T}^r_\varphi$, therefore (2.8) and Stokes’s theorem implies that the left hand side of (4.9) can be written also as $-\int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} \mathbf{T}_{\mu\nu}\zeta^\mu\chi^\nu d\theta d\varphi$, where $\mathbf{T}_{\mu\nu}$ is the Einstein–Hilbert energy-momentum tensor of the Dirac field given by (2.4). Thus the result (4.9) for $dM - \Omega_H dJ - \Phi_H dQ_e$ is completely analogous to the result obtained in the case of the scalar field in [16], and one can say that the conversion of a black hole into a naked singularity by a Dirac field is possible because the Einstein–Hilbert energy-momentum tensor of the Dirac field does not satisfy the null energy condition $\mathbf{T}^{\mu\nu}\chi_\mu\chi_\nu \geq 0$.

The case of combined scalar and electromagnetic test matter was also considered in Section 4.2 of [16], and also in that case it was found that $dM - \Omega_H dJ - \Phi_H dQ_e = -\int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} T_{\mu\nu}\zeta^\mu\chi^\nu d\theta d\varphi$, where $T_{\mu\nu}$ is the relevant Einstein–Hilbert energy-momentum tensor. This $T_{\mu\nu}$ satisfies the null energy condition, implying $d\eta \geq 0$. If the scalar field vanishes, then this case reduces to the case of purely electromagnetic test field.

The integrand on the right hand side of (4.11) can be expressed in a more explicit form. At the event horizon

$$\zeta^\mu \gamma^{\bar{0}} \gamma_\mu = \gamma^{\bar{0}} \gamma^r = \frac{-r_+^2}{\sqrt{2}(r_+^2 + a^2 \cos^2 \theta)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (4.14)$$

thus

$$\zeta_\mu \bar{\Psi} \gamma^\mu \Psi = \bar{\Psi} \gamma^r \Psi = \frac{-r_+^2}{\sqrt{2}(r_+^2 + a^2 \cos^2 \theta)} (|\Psi_1|^2 + |\Psi_4|^2), \quad (4.15)$$

where Ψ_1 and Ψ_4 denote the first and fourth components of Ψ . Furthermore,

$$\sqrt{-g} = (r^2 + a^2 \cos^2 \theta) \sin \theta, \quad (4.16)$$

therefore at the event horizon

$$\sqrt{-g} \zeta_\mu \bar{\Psi} \gamma^\mu \Psi = \frac{-r_\pm^2}{\sqrt{2}} \sin \theta (|\Psi_1|^2 + |\Psi_4|^2) . \quad (4.17)$$

These formulas hold for any spinor Ψ , regardless of the Dirac equation, thus they hold also for $\psi_{\omega,n}$. (4.11) can be rewritten therefore as

$$\begin{aligned} \int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} \hat{T}_{\mu\nu} \zeta^\mu \chi^\nu d\theta d\varphi &= \\ &= (2\pi)^2 \sum_n \int d\omega \int_H d\theta \frac{-r_\pm^2}{\sqrt{2}} \sin \theta (|(\psi_{\omega,n})_1|^2 + |(\psi_{\omega,n})_4|^2) \tilde{\omega} . \end{aligned} \quad (4.18)$$

Finally, for $d\eta < 0$ it is necessary that $(\psi_{\omega,n})_1$ or $(\psi_{\omega,n})_4$ be nonzero at the event horizon at least for some values of ω and n for which $\tilde{\omega} < 0$, therefore in principle it should be investigated if there is anything that could force $(\psi_{\omega,n})_1$ or $(\psi_{\omega,n})_4$ to be zero at the event horizon. If, invoking the separability of the Dirac equation (see B), it is assumed that $\psi_{\omega,n}$ is a linear combination of terms satisfying the ansatz (B.2), (B.11), (B.12), then $(\psi_{\omega,n})_1$ or $(\psi_{\omega,n})_4$ is nonzero at the event horizon if $R_+(r_+) \neq 0$ in these terms. ((B.2), (B.11) and (B.12) show that R_- does not enter $(\psi_{\omega,n})_1$ and $(\psi_{\omega,n})_4$.) As explained in more detail in B.1, $R_+(r_+)$ is not zero, therefore generally $(\psi_{\omega,n})_1$ and $(\psi_{\omega,n})_4$ do not have to be zero at the event horizon.

4.1 On possible backreaction effects

Regarding the question whether backreaction effects can be expected to prevent the formation of a naked singularity, it should be noted first that a result in rigorous test field approximation, which can be considered as a lowest order approximation, indicating the formation of a naked singularity is more conclusive than a result which indicates that a naked singularity is not formed (as in the cases of scalar and electromagnetic fields), because the latter type of result does not exclude the possibility of the formation of a naked singularity outside the domain of validity of the test field approximation, whereas the first type of result implies that the formation of a naked singularity may be avoided only if the perturbation is sufficiently large so that higher order effects can dominate. This also shows that considering higher order effects is more important when naked singularity formation is excluded at lowest order.

In the literature it has been emphasized that backreaction effects have to be taken into account properly, and that this usually restores the cosmic censor in scenarios in which it seems to be violated [52, 61, 62, 68, 69]. In these cases, in contrast with the case of the Dirac field, cosmic censorship is respected at lowest order, i.e. in rigorous test matter approximation, and the apparent violation of WCCC arises because effects beyond the lowest order are included in some way, but only partially. The restoration of cosmic censorship is achieved by properly taking into account all relevant effects. For example, in [52] the overspinning of a near extremal Reissner–Nordström black hole by waves carrying angular momentum was considered. Such

a setting immediately implies the inclusion of effects beyond lowest order, because η depends on J through J^2 , thus in the lowest order the η parameter of a Reissner–Nordström black hole cannot be changed by changing its angular momentum. In [52] it was shown that although an apparent violation of the WCCC can be found if the change of η due to the change of J is not neglected but the waves are assumed to propagate on fixed Reissner–Nordström background, cosmic censorship is restored if the effect of the waves on the background during the interaction process is also taken into account, as required by the consistency of the approximation applied. In [41] the overspinning of a slightly subextremal Kerr black hole with a test body was considered, and also in this study some higher order quantities were not neglected, while radiative and self-force effects were not taken into account. Later in [61, 62, 68, 69] it was argued that self-force effects are not negligible in this scenario and they might be the main effect preventing the violation of the WCCC.

In [54] a further interesting effect is described; the formation of another horizon outside a Reissner–Nordström black hole when a charged shell that would be expected to destroy it is adiabatically lowered towards its event horizon. This scenario is hard to compare with the case of the Dirac field, but even if a similar effect can show up also in the latter case, it is a higher order effect, therefore it cannot be expected to completely override the lowest order result. Regarding the adiabatic lowering of charged objects, it should also be noted that it is not necessary to assume that the particle comes from infinity in the derivations in [8, 9, 16] of the result that in rigorous test particle approximation the cosmic censorship principle is respected.

5 Superradiance

The setting in which the phenomenon of black hole superradiance occurs is similar to that of the thought experiment, with the difference that the initial black hole is not necessarily extremal and the quantity of interest is the total energy dE that flows through the event horizon, instead of $d\eta$. Superradiance occurs if dE has a sign that corresponds to an amplification of the energy of the field outside the event horizon. In addition, the angular momentum and the electric charge of the field can also be considered in the study of superradiance.

In the literature it is usual to describe superradiance in terms of the amplitude of suitable radial functions which arise when the complete separation of the variables is carried out (see e.g. [17, 18, 33]), but in this section we do not use these amplitudes, in accordance with our aim to avoid the use of the complete separation of variables as much as possible.

The superradiance of individual energy and angular momentum modes can also be defined. In this case the quantity that determines if a certain mode is superradiant is the sign of the rate $\frac{dE}{d\tau}$.

5.1 Absence of superradiance of Dirac fields

If some matter described by the Dirac field is thrown into a black hole, then the total electric charge absorbed by the black hole is

$$dQ = \int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} e j^r d\theta d\varphi . \quad (5.1)$$

By definition $j^r = \zeta_\mu j^\mu$, and one can argue, in the same way as in Section 4, that at the event horizon ζ^μ is past directed and null, j^μ is always future directed and null or time-like, thus $j^r \leq 0$, and so $e dQ < 0$. This means that the total electric charge falling through the event horizon always has the same sign as the charge of the Dirac field, thus the charge outside the event horizon does not increase. In other words, the Dirac field does not show superradiance in relation to electric charge.

Considering energy and angular momentum, using the Fourier expansion (4.10) one finds that the total energy and angular momentum absorbed by the black hole is

$$\begin{aligned} dE &= - \int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} \mathcal{E}^r d\theta d\varphi \\ &= (2\pi)^2 \sum_n \int d\omega \int_H d\theta \sqrt{-g} (-\omega \bar{\psi}_{\omega,n} \gamma^r \psi_{\omega,n}) \end{aligned} \quad (5.2)$$

$$\begin{aligned} dL &= \int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} \mathcal{J}^r d\theta d\varphi \\ &= (2\pi)^2 \sum_n \int d\omega \int_H d\theta \sqrt{-g} (-n \bar{\psi}_{\omega,n} \gamma^r \psi_{\omega,n}) . \end{aligned} \quad (5.3)$$

In the derivation of (5.2) and (5.3) it is useful to write (4.2) and (4.4) as $\mathcal{E}^\mu = \frac{1}{2}(\bar{\Psi} i \gamma^\mu \partial_t \Psi - \partial_t \bar{\Psi} i \gamma^\mu \Psi)$ and $\mathcal{J}^\mu = \frac{1}{2}(\bar{\Psi} i \gamma^\mu \partial_\varphi \Psi - \partial_\varphi \bar{\Psi} i \gamma^\mu \Psi) - e Q_m C j^\mu$, because the vector potential does not appear explicitly in the latter expressions.

Since, as explained in Section 4, $\bar{\psi}_{\omega,n} \gamma^r \psi_{\omega,n} \leq 0$ at the event horizon, the integrands in (5.2) and (5.3) have the same signs as ω and n , respectively. Consequently, if the Fourier expansion of Ψ contains only positive frequency modes, then the energy falling through the event horizon is also positive and the energy outside the event horizon does not increase, and analogous statements can be made for negative frequency modes and for angular momentum. This means that the Dirac field is not superradiant in relation to energy and angular momentum either.

Instead of considering solutions of the form (4.10), one can consider waves consisting of a single mode, $e^{-i\omega\tau} e^{i(n - C e Q_m)\varphi} \psi_{\omega,n}(r, \theta)$. dQ , dE and dL are not meaningful for such waves, but one can study the rates $\frac{dQ}{d\tau} = \int_H \sqrt{-g} e j^r d\theta d\varphi$, $\frac{dE}{d\tau} =$

$-\int_H \sqrt{-g} \mathcal{E}^r d\theta d\varphi$, $\frac{dL}{d\tau} = \int_H \sqrt{-g} \mathcal{J}^r d\theta d\varphi$. These rates can be expressed as

$$\frac{dQ}{d\tau} = 2\pi \int_H d\theta \sqrt{-g} (e \bar{\psi}_{\omega,n} \gamma^r \psi_{\omega,n}) \quad (5.4)$$

$$\frac{dE}{d\tau} = 2\pi \int_H d\theta \sqrt{-g} (-\omega \bar{\psi}_{\omega,n} \gamma^r \psi_{\omega,n}) \quad (5.5)$$

$$\frac{dL}{d\tau} = 2\pi \int_H d\theta \sqrt{-g} (-n \bar{\psi}_{\omega,n} \gamma^r \psi_{\omega,n}) . \quad (5.6)$$

Using these expressions one can argue in the same way as above that the Dirac field does not have superradiant modes.

5.2 Superradiant frequency range of scalar fields

The massive complex scalar field has the Lagrangian density

$$\mathcal{L} = g^{\mu\nu} (\partial_\mu - ieA_\mu) \Phi^* (\partial_\nu + ieA_\nu) \Phi - m^2 \Phi^* \Phi \quad (5.7)$$

and the corresponding field equation $(\nabla^\mu + ieA^\mu)(\nabla_\mu + ieA_\mu)\Phi = -m^2\Phi$. The electric current of the scalar field is

$$j^\mu = \frac{\partial \mathcal{L}}{\partial A_\mu} = -ie[\Phi^*(\partial^\mu + ieA^\mu)\Phi - \Phi(\partial^\mu - ieA^\mu)\Phi^*] . \quad (5.8)$$

In [16] we found the energy and angular momentum current densities \mathcal{E}^μ and \mathcal{J}^μ by applying Noether's theorem. The result for \mathcal{J}^μ had to be modified in the same way as in Section 4 to eliminate its dependence on the gauge parameter C . \mathcal{E}^μ and \mathcal{J}^μ are given by the expressions

$$\mathcal{E}^\mu = \hat{T}^\mu{}_\tau - A_\tau j^\mu , \quad \mathcal{J}^\mu = \hat{T}^\mu{}_\varphi - (A_\varphi - Q_m C) j^\mu , \quad (5.9)$$

where

$$\hat{T}_{\mu\nu} = (\partial_\mu - ieA_\mu) \Phi^* (\partial_\nu + ieA_\nu) \Phi + (\partial_\mu + ieA_\mu) \Phi (\partial_\nu - ieA_\nu) \Phi^* - g_{\mu\nu} \mathcal{L} . \quad (5.10)$$

Using the Fourier expansion

$$\Phi = \sum_n \int d\omega e^{-i\omega\tau} e^{i(n-CeQ_m)\varphi} \phi_{\omega,n}(r, \theta) \quad (5.11)$$

of Φ , where $e^{-i\omega\tau}e^{i(n-CeQ_m)\varphi}\phi_{\omega,n}(r,\theta)$ are solutions of the field equation, one obtains the following results for dQ , dE and dL :

$$\begin{aligned} dQ &= -\int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} j^r d\theta d\varphi \\ &= -(2\pi)^2 \sum_n \int d\omega \int_H d\theta 2(a^2 + r_+^2) \sin\theta \phi_{\omega,n}^* \phi_{\omega,n} e\tilde{\omega} \end{aligned} \quad (5.12)$$

$$\begin{aligned} dE &= -\int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} \mathcal{E}^r d\theta d\varphi \\ &= (2\pi)^2 \sum_n \int d\omega \int_H d\theta 2(a^2 + r_+^2) \sin\theta \phi_{\omega,n}^* \phi_{\omega,n} \omega\tilde{\omega} \end{aligned} \quad (5.13)$$

$$\begin{aligned} dL &= \int_{-\infty}^{\infty} d\tau \int_H \sqrt{-g} \mathcal{J}^r d\theta d\varphi \\ &= (2\pi)^2 \sum_n \int d\omega \int_H d\theta 2(a^2 + r_+^2) \sin\theta \phi_{\omega,n}^* \phi_{\omega,n} n\tilde{\omega} , \end{aligned} \quad (5.14)$$

where $\tilde{\omega}$ is defined as in (4.12). In the derivation of (5.12), (5.13) and (5.14) only the derivatives $\partial_\tau\Phi$ and $\partial_\varphi\Phi$ of Φ appear, which can be evaluated easily, and one can also use (3.10)-(3.12) and (4.17). Due to the factor $g_{\mu\nu}$, the $-g_{\mu\nu}\mathcal{L}$ term appearing in (5.10) does not give any contribution to dE and dL , as $\delta_\tau^r = \delta_\varphi^r = 0$. For $e^{-i\omega\tau}e^{i(n-CeQ_m)\varphi}\phi_{\omega,n}(r,\theta)$ to be single valued for both $C = 1$ and $C = -1$, both $n - eQ_m$ and $n + eQ_m$ have to be integer, implying that n and eQ_m are either integer or half-integer, thus in the summations n should take integer values if eQ_m is integer and half-integer values if eQ_m is half-integer [73]. It should also be noted that far from the black hole only modes with $|\omega| > m$ describe propagating waves.

Since $\phi_{\omega,n}^*\phi_{\omega,n}$ is positive unless $\phi_{\omega,n} = 0$, (5.13) shows that dE is negative if the main contribution to the integral comes from the frequency range where

$$\omega\tilde{\omega} < 0 . \quad (5.15)$$

In this case the scalar field exhibits superradiance in the sense that the total energy of the field outside the event horizon increases. The sign of the integrands in (5.12) and (5.14) is also determined by $e\tilde{\omega}$ and $n\tilde{\omega}$, respectively, instead of $e\omega$ and $n\omega$.

For individual modes $e^{-i\omega\tau}e^{i(n-CeQ_m)\varphi}\phi_{\omega,n}(r,\theta)$, the rates $\frac{dE}{d\tau}$, $\frac{dL}{d\tau}$ and $\frac{dQ}{d\tau}$ can be expressed as

$$\frac{dQ}{d\tau} = -2\pi \int_H d\theta 2(a^2 + r_+^2) \sin\theta \phi_{\omega,n}^* \phi_{\omega,n} e\tilde{\omega} \quad (5.16)$$

$$\frac{dE}{d\tau} = 2\pi \int_H d\theta 2(a^2 + r_+^2) \sin\theta \phi_{\omega,n}^* \phi_{\omega,n} \omega\tilde{\omega} \quad (5.17)$$

$$\frac{dL}{d\tau} = 2\pi \int_H d\theta 2(a^2 + r_+^2) \sin\theta \phi_{\omega,n}^* \phi_{\omega,n} n\tilde{\omega} . \quad (5.18)$$

Of course, the superradiant frequencies following from (5.17) are the same as those that follow from (5.13).

6 Conclusion

We showed that a dyonic Kerr–Newman black hole can be converted into a naked singularity by interaction with massive charged classical Dirac fields, generalizing a recent result [15] which applies to massless Dirac fields and Kerr black holes. We found that for this conversion the spectrum of the Dirac field has to be dominated by modes with temporal and angular frequencies ω and n satisfying the inequality (4.12). We also showed that the creation of a naked singularity is possible because the null energy condition is not satisfied by the Einstein–Hilbert energy-momentum tensor of the Dirac field. This means that the classical Dirac field does not satisfy the criteria under which the WCCC is expected to hold, thus in strict sense the possibility found in [15] and in the present paper does not contradict the WCCC. These features of the Dirac field are complementary to those of the scalar and the electromagnetic field, which satisfy the null energy condition and as a consequence cannot convert a black hole into a naked singularity.

We gave a derivation of the absence of superradiance of the Dirac field around dyonic Kerr–Newman black holes, and we determined the temporal and angular frequencies for which the scalar field is superradiant. We found that the superradiant modes are those that satisfy the condition (5.15). The frequencies satisfying (4.12) partially agree with those satisfying (5.15).

Although the well-known separability of the scalar wave equation and of the Dirac equation around black holes is very useful for several purposes and is often used also in the discussion of the testing of the WCCC or of the superradiance phenomenon, its use could be largely avoided in this paper. Nevertheless, as we used it in an argument at the end of Section 4, we described it in an appendix in horizon-penetrating coordinates.

In the derivations the test field approximation was applied, but we argued that the destruction of black holes cannot be completely prevented by backreaction effects. It was also assumed that the final state that arises after the interaction of the black hole and the Dirac field is again a dyonic Kerr–Newman configuration, without any Dirac hair. This assumption is in accordance with the no-hair conjecture, but its validity would nevertheless be interesting to investigate.

It is natural to hope that the formation of a naked singularity can be ruled out in quantum theory, since problems related to negative energies are absent in quantum field theory in Minkowski spacetime. Results on quantum effects have already appeared in the literature [31, 47, 48, 49, 72], and comments can be found also in [15]. We leave further investigations of the quantum Dirac field for future work.

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A Dirac spinor fields in curved spacetime

In this paper we apply the tetrad formalism to incorporate Dirac spinor fields into general relativity, as described e.g. in [80].

We denote internal Lorentz vector indices by letters with an overbar. In contrast with Lorentz and spacetime vector indices, the normal position of Dirac spinor indices is taken to be the lower one, and hence cospinor indices are in the upper position. Dirac spinor indices are often not written out explicitly.

Lorentz vector and spinor indices are scalar indices with respect to spacetime diffeomorphisms; in particular, tensors having only Lorentz vector and spinor indices are scalars under spacetime diffeomorphisms. On the other hand, local Lorentz transformations do not act on the spacetime manifold and spacetime vector indices are scalar indices with respect to them.

We use orthonormal tetrad fields $V_\mu^{\bar{\mu}}$, i.e. $g^{\bar{\mu}\bar{\nu}} = V_\mu^{\bar{\mu}} V_\nu^{\bar{\nu}} g^{\mu\nu}$, $g^{\bar{\mu}\bar{\nu}} = \text{diag}(1, -1, -1, -1)$.

The Levi-Civita covariant differentiation is extended to tensors with Lorentz vector indices in the following way:

$$\nabla_\mu v^{\bar{\nu}} = \partial_\mu v^{\bar{\nu}} + \mathcal{S}_{\bar{\lambda}\mu}^{\bar{\nu}} v^{\bar{\lambda}}, \quad \mathcal{S}_{\bar{\eta}\mu}^{\bar{\lambda}} = -V_{\bar{\eta}}^\nu \nabla_\mu^{LC} V_\nu^{\bar{\lambda}}, \quad (\text{A.1})$$

where $\mathcal{S}_{\bar{\eta}\mu}^{\bar{\lambda}}$ is an analogue of the Christoffel symbols. On the right hand side the superscript LC indicates that the Levi-Civita covariant differentiation should be used. With this definition the covariant derivative of the Lorentz metric tensor and of the tetrad field is zero: $\nabla_\mu g_{\bar{\nu}\bar{\lambda}} = 0$, $\nabla_\mu V_\nu^{\bar{\lambda}} = 0$.

The Dirac gamma matrices γ^μ are defined as

$$\gamma^\mu = V_\mu^{\bar{\mu}} \gamma^{\bar{\mu}}, \quad (\text{A.2})$$

where $\gamma^{\bar{\mu}}$ are the standard Minkowski space gamma matrices. We use the Weyl representation for them,

$$\gamma^{\bar{0}} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^{\bar{i}} = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad i = 1, 2, 3, \quad (\text{A.3})$$

where I denotes the 2×2 identity matrix and σ^i are the Pauli sigma matrices (see [81]). γ^μ has the property $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, where $\{, \}$ denotes the anticommutator $\{A, B\} = AB + BA$.

The Levi-Civita connection can be extended to tensors having Dirac spinor indices in the following way:

$$\nabla_\mu \psi_\alpha = \partial_\mu \psi_\alpha + S_\alpha^{\beta}{}_\mu \psi_\beta, \quad (\text{A.4})$$

where

$$S_\mu = \frac{1}{4} \sigma^{\bar{\nu}\bar{\lambda}} \mathcal{S}_{\bar{\nu}\bar{\lambda}\mu}, \quad \sigma^{\bar{\mu}\bar{\nu}} = \frac{1}{2} [\gamma^{\bar{\mu}}, \gamma^{\bar{\nu}}]. \quad (\text{A.5})$$

With this definition of the covariant differentiation of spinors the covariant derivative of the gamma tensor is zero: $\nabla_\mu \gamma^\nu = 0$.

In the Weyl representation the Dirac conjugation for Dirac spinors with a lower spinor index is customarily expressed as $\bar{\psi} = \psi^{*T} \gamma^{\bar{0}}$, where the T denotes transposition, the * denotes componentwise complex conjugation, and as usual the Dirac conjugation is denoted by an overbar.

Finally we note that we do not introduce any raising and lowering convention for spinor indices.

B Separation of variables for the Dirac equation

In this appendix the separation of variables for the Dirac equation in dyonic Kerr–Newman background is described. This is done in the horizon-penetrating coordinates $(\tau, r, \theta, \varphi)$, using the Kinnersley-type tetrad $\tilde{V}_{\mu}^{\bar{\mu}}$ introduced in Section 3.3 and the Weyl representation for the Dirac gamma matrices described in A. The separability of the Dirac equation in dyonic Kerr–Newman background was shown first in [37], in Boyer–Lindquist coordinates. The reader is referred to this article and to [18, 19] for further references to earlier results. The recent article [30] focuses on the separability of the Dirac equation in horizon-penetrating coordinates, but only in Kerr geometry.

The asymptotic behaviour at the event horizon of the solutions of the radial equations that arise after the separation of variables is also discussed in a subsection.

The application of the method of separation of variables to finding solutions of the Dirac equation in dyonic Kerr–Newman background begins with assuming that Ψ depends on τ and φ harmonically as

$$\Psi(\tau, r, \theta, \varphi) = e^{-i\omega\tau} e^{i(n - CeQ_m)\varphi} \psi(r, \theta), \quad (\text{B.1})$$

where ω and n denote the temporal and angular frequency, respectively. The term $-CeQ_m$ in the factor $e^{i(n - CeQ_m)\varphi}$ is included because of the gauge transformation done at equation (3.8) in Section 3.1 (see also [37]). After introducing the new field variables f_i , $i = 1, \dots, 4$, as

$$f_1 = \psi_1, \quad f_2 = \frac{1}{r}(r - ia \cos \theta)\psi_2, \quad f_3 = \frac{1}{r}(r + ia \cos \theta)\psi_3, \quad f_4 = \psi_4, \quad (\text{B.2})$$

where ψ_i , $i = 1, \dots, 4$, denote the components of ψ , the Dirac equation can be written in the form

$$D_+ f_3 + L_+ f_4 = -im(r + ia \cos \theta) f_1 \quad (\text{B.3})$$

$$D_- f_4 + L_- f_3 = -im(r + ia \cos \theta) f_2 \quad (\text{B.4})$$

$$D_- f_1 - L_+ f_2 = -im(r - ia \cos \theta) f_3 \quad (\text{B.5})$$

$$D_+ f_2 - L_- f_1 = -im(r - ia \cos \theta) f_4, \quad (\text{B.6})$$

where

$$D_+ = \frac{\sqrt{2}}{r^2}[-Mr + r^2 + i(2anr - 2eQ_e r^2 - 2a^2 r \omega - 2r^3 \omega + r \Delta \omega) + r \Delta \partial_r] \quad (\text{B.7})$$

$$D_- = \frac{-1}{\sqrt{2}}(1 + i r \omega + r \partial_r) \quad (\text{B.8})$$

$$L_+ = \frac{1}{2 \sin \theta}[-2n + (1 + 2eQ_m) \cos \theta + a \omega(1 - \cos 2\theta)] + \partial_\theta \quad (\text{B.9})$$

$$L_- = \frac{1}{2 \sin \theta}[2n + (1 - 2eQ_m) \cos \theta - a \omega(1 - \cos 2\theta)] + \partial_\theta. \quad (\text{B.10})$$

We note that in (B.2) the factors in front of ψ_2 and ψ_3 are chosen so that in the $r \rightarrow \infty$ limit $f_2 \rightarrow \psi_2$ and $f_3 \rightarrow \psi_3$.

By taking the ansatz

$$f_1(r, \theta) = R_+(r)S_+(\theta) \quad f_2(r, \theta) = R_-(r)S_-(\theta) \quad (\text{B.11})$$

$$f_3(r, \theta) = R_-(r)S_+(\theta) \quad f_4(r, \theta) = R_+(r)S_-(\theta) \quad (\text{B.12})$$

the r and θ parts of equations (B.3)-(B.6) become separated and one finds the ordinary differential equations

$$D_+ R_- - (\lambda - i m r) R_+ = 0 \quad (\text{B.13})$$

$$D_- R_+ + (\lambda + i m r) R_- = 0 \quad (\text{B.14})$$

$$L_+ S_- + (\lambda - m a \cos \theta) S_+ = 0 \quad (\text{B.15})$$

$$L_- S_+ - (\lambda + m a \cos \theta) S_- = 0 \quad (\text{B.16})$$

for R_+ , R_- , S_+ and S_- , where λ is the separation constant. Initially one introduces different separation constants in each equation (B.3)-(B.6), but then one sees that they have to be related.

By eliminating R_+ or R_- and S_+ or S_- one gets the second order decoupled equations

$$D_- D_+ R_- - \frac{i m r}{\sqrt{2}(\lambda - i m r)} D_+ R_- + (\lambda^2 + m^2 r^2) R_- = 0 \quad (\text{B.17})$$

$$D_+ D_- R_+ - \frac{i \sqrt{2} m \Delta}{r(\lambda + i m r)} D_- R_+ + (\lambda^2 + m^2 r^2) R_+ = 0 \quad (\text{B.18})$$

and

$$L_- L_+ S_- - \frac{m a \sin \theta}{\lambda - m a \cos \theta} L_+ S_- + (\lambda^2 - m^2 a^2 \cos^2 \theta) S_- = 0 \quad (\text{B.19})$$

$$L_+ L_- S_+ + \frac{m a \sin \theta}{\lambda + m a \cos \theta} L_- S_+ + (\lambda^2 - m^2 a^2 \cos^2 \theta) S_+ = 0. \quad (\text{B.20})$$

It is useful to write out the explicit form of L_-L_+ , L_+L_- , D_-D_+ , D_+D_- :

$$\begin{aligned}
L_-L_+S_- &= -\frac{1}{8\sin^2\theta} \left[3 + 8n^2 + 8eQ_m + 4e^2Q_m^2 - 8an\omega + 3a^2\omega^2 \right. \\
&\quad - 2(n(4 + 8eQ_m) + a(1 - 2eQ_m)\omega) \cos\theta \\
&\quad + (-1 + 4e^2Q_m^2 + 8an\omega - 4a^2\omega^2) \cos 2\theta \\
&\quad \left. + (2a\omega - 4aeQ_m\omega) \cos 3\theta + a^2\omega^2 \cos 4\theta \right] S_- \\
&\quad + \frac{\cos\theta}{\sin\theta} \partial_\theta S_- + \partial_\theta^2 S_- , \tag{B.21}
\end{aligned}$$

$$\begin{aligned}
L_+L_-S_+ &= -\frac{1}{8\sin^2\theta} \left[3 + 8n^2 - 8eQ_m + 4e^2Q_m^2 - 8an\omega + 3a^2\omega^2 \right. \\
&\quad + 2(n(4 - 8eQ_m) + a(1 + 2eQ_m)\omega) \cos\theta \\
&\quad + (-1 + 4e^2Q_m^2 + 8an\omega - 4a^2\omega^2) \cos 2\theta \\
&\quad \left. - (2a\omega + 4aeQ_m\omega) \cos 3\theta + a^2\omega^2 \cos 4\theta \right] S_+ \\
&\quad + \frac{\cos\theta}{\sin\theta} \partial_\theta S_+ + \partial_\theta^2 S_+ , \tag{B.22}
\end{aligned}$$

and

$$\begin{aligned}
D_+D_-R_+ &= \left[-1 + 2ieQ_e + \frac{M - 2ian}{r} + \omega(iM + ir + 2an - 2eQ_e r) \right. \\
&\quad \left. + \frac{2i\omega(a^2 - \Delta)}{r} + \omega^2(\Delta - 2a^2 - 2r^2) \right] R_+ \\
&\quad + \left[M - r - \frac{2\Delta}{r} + 2i(-an + eQ_e r) + 2i\omega(a^2 + r^2 - \Delta) \right] \partial_r R_+ \\
&\quad - \Delta \partial_r^2 R_+ , \tag{B.23}
\end{aligned}$$

$$\begin{aligned}
D_-D_+R_- &= \left[-1 + 2ieQ_e + \omega(3iM + ir + 2an - 2eQ_e r) \right. \\
&\quad \left. + \omega^2(\Delta - 2a^2 - 2r^2) \right] R_- \\
&\quad + \left[3(M - r) + 2i(-an + eQ_e r) + 2i\omega(a^2 + r^2 - \Delta) \right] \partial_r R_- \\
&\quad - \Delta \partial_r^2 R_- . \tag{B.24}
\end{aligned}$$

If the mass of the Dirac field is zero, then after multiplying by the integrating factor $\sin\theta$ the angular equations (B.19) and (B.20) take a Sturm–Liouville form. The weight factor appearing in the scalar product for the solutions is also $\sin\theta$. Although the case $m \neq 0$ is more complicated, it was studied e.g. in [25, 26, 27, 28, 29] at $Q_m = 0$.

In the $r \rightarrow \infty$ limit the radial equations (B.17) and (B.18) become

$$-\omega^2 R_\pm - \partial_r^2 R_\pm + m^2 R_\pm = 0 . \tag{B.25}$$

B.1 Asymptotic behaviour of the radial functions at the event horizon

The second order radial equations (B.17) and (B.18) are singular at the event horizon; the singularity is regular if the black hole is not extremal and irregular if the black hole is extremal. By analyzing the asymptotic behaviour of the solutions near the event horizon one finds two kinds of asymptotic behaviour. One of them is such that R_+ (or R_-) approaches a finite nonzero value as $r \rightarrow r_+$, the other one is such that R_+ goes to zero and $|R_-|$ to infinity as $r \rightarrow r_+$. Solutions with the latter behaviour can be considered unphysical.

More specifically, in the non-extremal case R_+ takes the form

$$c_1(r - r_+)^{s_1}y_1(r - r_+) + c_2(r - r_+)^{s_2}y_2(r - r_+) , \quad (\text{B.26})$$

where c_1 and c_2 are integration constants, y_1 and y_2 are functions that are regular and nonzero at 0, and the characteristic exponents s_1 and s_2 are solutions of the indicial equation

$$s(s - 1) - \frac{1}{r_+ - r_-}[M - r_+ + 2i(a^2 + r_+^2)\tilde{\omega}]s = 0 . \quad (\text{B.27})$$

The solutions of this equation are

$$s_1 = 0, \quad s_2 = \frac{1}{2} + i\frac{2(a^2 + r_+^2)\tilde{\omega}}{r_+ - r_-} , \quad (\text{B.28})$$

thus the solution corresponding to $c_2 = 0$ is regular and nonzero at the event horizon, whereas the solution corresponding to $c_1 = 0$ is not regular but is vanishing at r_+ . The latter solution can also be written as

$$e^{s_2 \log(r - r_+)} y_2(r - r_+) , \quad (\text{B.29})$$

showing that near the event horizon the absolute value of this solution behaves like $\sim (r - r_+)^{1/2}$ and the oscillation frequency of its phase increases to infinity as $r \rightarrow r_+$.

In the extremal case the asymptotic behaviour of the $R_+ \rightarrow 0$ type solution of (B.18) at the event horizon is found to be

$$\sim \exp \left[-\frac{2i(a^2 + r_+^2)\tilde{\omega}}{r - r_+} + (1 + 2ieQ_e + 4i\omega r_+) \log(r - r_+) \right] , \quad (\text{B.30})$$

showing that the absolute value of this solution behaves like $\sim (r - r_+)$ and the oscillation frequency of its phase increases to infinity as $r \rightarrow r_+$.

The characteristic exponents for R_- in the non-extremal case are

$$s_1 = 0, \quad s_2 = -\frac{1}{2} + i\frac{2(a^2 + r_+^2)\tilde{\omega}}{r_+ - r_-} , \quad (\text{B.31})$$

as can be expected from (B.14) and (B.28). Similarly, in the extremal case the asymptotic behaviour of the $|R_-| \rightarrow \infty$ type solution of (B.17) at the event horizon is

$$\sim \exp \left[-\frac{2i(a^2 + r_+^2)\tilde{\omega}}{r - r_+} + (-1 + 2ieQ_e + 4i\omega r_+) \log(r - r_+) \right] . \quad (\text{B.32})$$

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